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December 7, 1944

Dr. Hsue-shen Tsien
California Institute of Technology
Pasadena, Calif.

Dear Dr. Tsien:

Enclosed we are sending first results concerning the problem of an airfoil of minimum drag in supersonic flow.

I should not have overlooked the fact that the chord enters the definition of lift-and drag-coefficient. In the present formulation the problem is accessible and has a well-defined solution under certain assumptions.

The assumptions we have made are:

I. The foil has a sharp leading tip (or edge) at which shock fronts develop behind which the flow is still supersonic.

II. The flow past the leading shock front is a simple rarefaction ~~vane~~ wave (in the terminology adopted in our Manual). To formulate this condition in a different manner one may describe the flow past the leading shock front as follows: A rarefaction ~~vane~~ wave originates at the profile and (at least partly) overtakes the shock front. The major effect of this interaction is that rarefaction - and shock ~~vane~~ wave - gradually cancel each other; the minor effect is that a ~~vane~~ wave is reflected. This reflected ~~vane~~ wave (or part of it) hits the profile, is again reflected and so forth. The assumption can then be thus formulated: the ~~vane~~ wave reflected at the shock front is ignored. This assumption seems well-justified if the Mach number of the flow behind the leading shock front is appreciably greater than 1. (Experience in similar problems at least indicates that such interaction of a rarefaction ~~vane~~ wave overtaking a shock ~~vane~~ wave is very slow and involves only weak reflections.)

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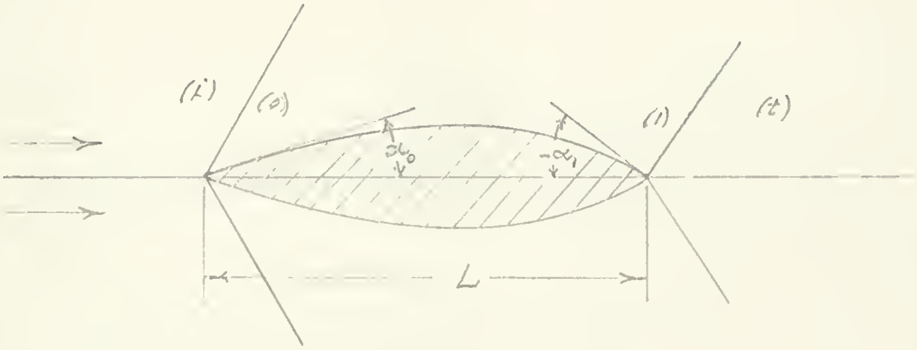
III. The foil has a sharp trailing tip (at which again shock fronts develop). This assumption cannot be maintained throughout, in view of the possibility of earlier jet detachment. More about this problem later.

We have first considered the case that the prescribed lift coefficient is zero and that the profile is symmetrical. The problem has then a well-defined unique solution if the thickness ratio does not exceed a certain value (depending on the Mach number of the oncoming flow). The profile begins with a short straight section, then turns continuously. It is interesting to note that the wedge angle of the trailing tip is greater than that at the leading tip. Thus the profile looks as in the diagram.



To determine these minimum profiles requires much labor. We have determined instead slightly different profiles, which do not have a straight section at the tip. I think that this deviation is within the limits of accuracy implied by condition II and I suspect that the correct minimum profile derived without imposing condition II has no such straight section at the tip.

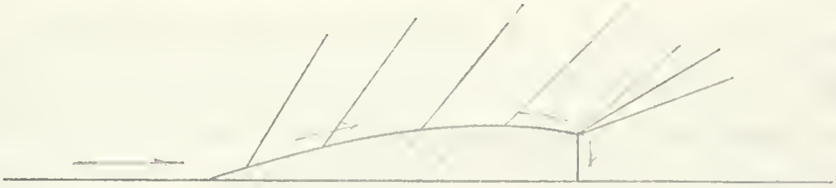
In detail we have calculated significant quantities for the case that the Mach number of the flow opposing the profile is

$$M_1 = 1.6$$

We then have plotted as a function of the thickness ratio A/L^2 the drag coefficient C_D (the minimum drag is slightly smaller, as mentioned before); the angles α_0 and $-\alpha_1$ at leading and trailing tip; the ratios p_0/p_1, p_1/p_1, p_t/p_1 of pressures behind leading shock, in front of and behind trailing shock to the pressure p_1 in front of the profile.

We should like to learn from you which range of values for M_1, A/L^2, C_L would be of particular interest.

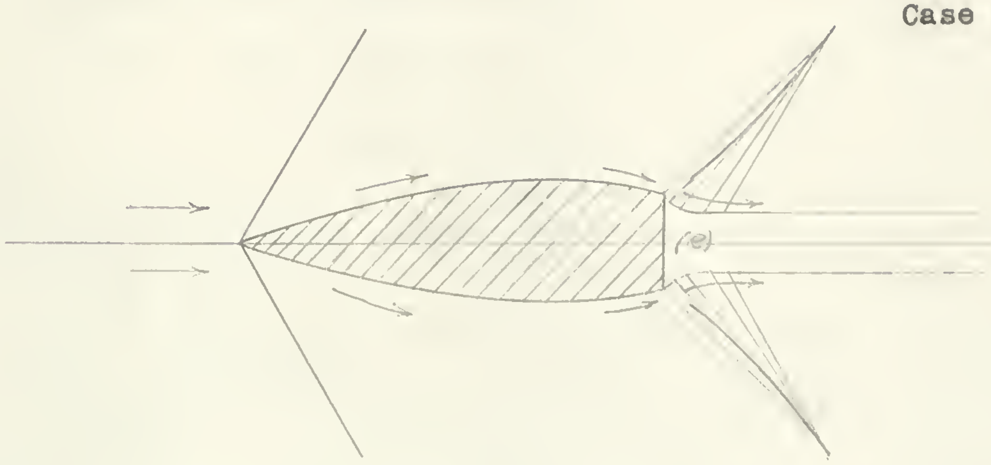
When the thickness ratio A/L^2 exceeds a certain value, no minimum drag profile exists that satisfies I, II, III. A minimizing sequence of such profiles approaches a profile that possesses a flat end section (as shown in the diagram).



Condition II, which is satisfied by this limiting profile, implies that the flow slides along the flat section in vertical direction, the pressure there (it may be called p_∞) being determined accordingly. Of course, this is a completely unreal situation. Condition III must therefore be modified. Instead then we impose Condition III′: a flat end section is admitted. The flow is not assumed to slide along this section. Instead the pressure at this section (p_e) is prescribed.

Then either the minimum profile satisfies III (Case A) and the flow is as before or the minimum flow is as in the following diagram (Case B). The rarefaction ~~vane~~ wave at the corner turns the flow only until the pressure p_e is reached.

Case B



In an enclosed diagram we show the graph of the maximum value $\bar{p}_e$ of the pressure p_e which may be prescribed at the back end without necessitating a back end section. In other words, if $p_e < \bar{p}_e$ is prescribed, condition III is satisfied automatically by the minimum profile for which only III was required (Case A). For $p_e > \bar{p}_e$ we have Case B. Already for the rather small thickness ratio A/L^2 = ~~.050~~ .035 the value $\bar{p}_e$ is less than 1/2.

The question then arises which should be the value of the prescribed pressure (or rather average pressure) p_e at the flat end section. (Clearly p_e should be larger than p_∞; otherwise the same awkward situation as before would arise). It seems likely that a value such as

$$p_e = \frac{1}{2} p_1$$

(p_1 being the initial pressure of 1 atm., say), is reasonable. I have been told that an analysis of the drag of projectiles has shown that it is about double the value obtained from the Karman-Moore theory and adding the skin friction contribution. Thus 1/2 of the drag is due to the underpressure at the flat back. This then would indicate a pressure of 1/2 atm. there. I have, however, no idea what actually determines the value of this pressure. Which value would you suggest us to take? Photographs indicate that the

wake contracts; that the flow outside the wake turns toward the direction of the main stream and creates a shock ~~vane~~ wave which cuts through the rarefaction ~~vane~~ wave developing from the corner. This flow at the same time slows down and the pressure in it approaches the value p_1 at ∞. Also in the wake the pressure rises from the average value p_e to p_1. If the wake consisted of quiet air which followed the body, the pressure would be constant in it. Therefore the pressure rise in the wake results from the fact that vorticity created at the profile and the wake boundary is continually shed into the wake. From the theory of the Karman Street one should expect a drag contribution. I do not see, however, how this should operate in detail. For subsonic flow one could say that the stagnation pressure, which is essentially assumed at the back of the body, is lower than that in front due to the action of viscosity. I do not notice, however, how such an argument could be applied to supersonic flow to explain the occurrence of a sub-atmospheric pressure at the back.

There is another reason why the flow pattern assumed in condition III might not be correct under certain circumstances. The pressure p_t behind the trailing shock must eventually adjust itself to the value p_1 at infinity. This, I imagine can be done by the faint waves that result from reflection at the leading shock front and crossing the trailing shock front. If the terminal pressure p_t is rather low, I feel that adjustment to p_1 takes place in a difference manner, viz. by detachment of the flow (combined with a shock) before the rear tip is reached. This would then correspond to the situation in nozzle flow where detachment takes place inside the nozzle when the shockless flow would have too low a pressure at the exit. From Stanton's and Frazer's experiments I took it that detachment takes place when the pressure has reached the neighborhood of 1/2 atm. Should one assume a similar value in the present problem?

For the Mach number $M_1 = 1.6$ the pressure p_1 in front of the trailing shock front remains above .~~74~~ .55 p_1 when the thickness ratio A/c^2 is below ~~.075~~ .06. (In the latter case we are certainly already in Case B where the drag can be lowered by a flat end section.)

[redacted]

There is clearly another case in which flow and shock front would detach before the rear tip, viz. when the rear angle ($-\alpha_1$) is so large that no adjustment to parallel flow through a shock is possible. For $M_1 = 1.6$ $A/\ell^2 \leq \overset{.06}{\cancel{.073}}$ this situation is not reached.

I do not see how this detachment could occur in connection with the phenomenon of limiting line. (A separate letter written before this one contains a few remarks about this problem.)

The theory on which our calculations were based is not yet in a sufficiently clear shape. As soon as it is ready, I shall send it to you.

I should appreciate it if you could make your comments on the basis of the remarks made in this letter.

We have looked into the matter of flutter in supersonic flight. v. Borhelz has apparently only calculated the resulting forces and moments without developing a flutter theory such as that of Theodorsen. I therefore have been unable to draw any significant conclusions from his results.

I am not quite clear about the aim of any further investigation in this field. Was your idea that we should do the same as v. Borhelz but go to the next order in the expansion?

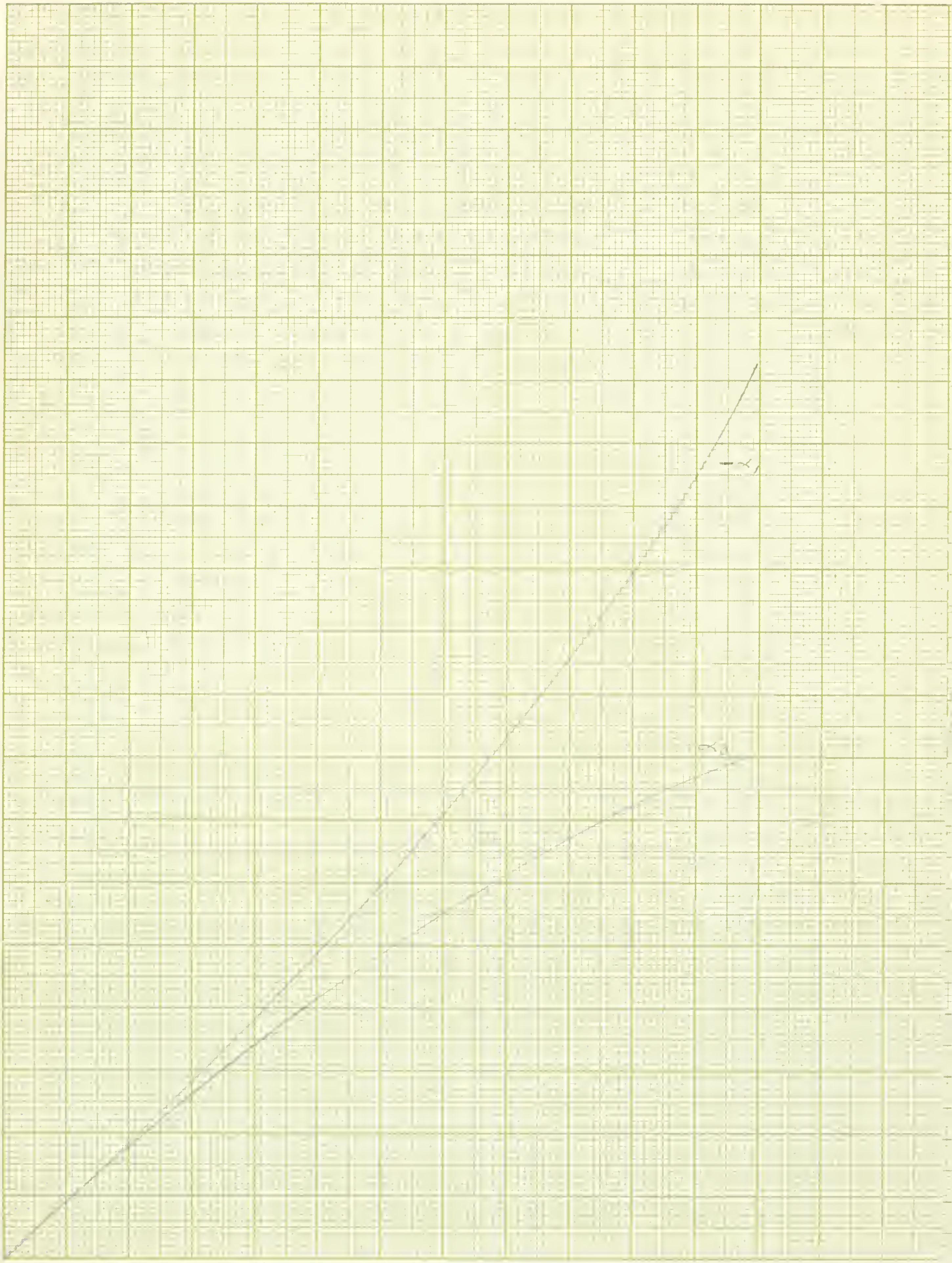
I should like to hear from you soon.

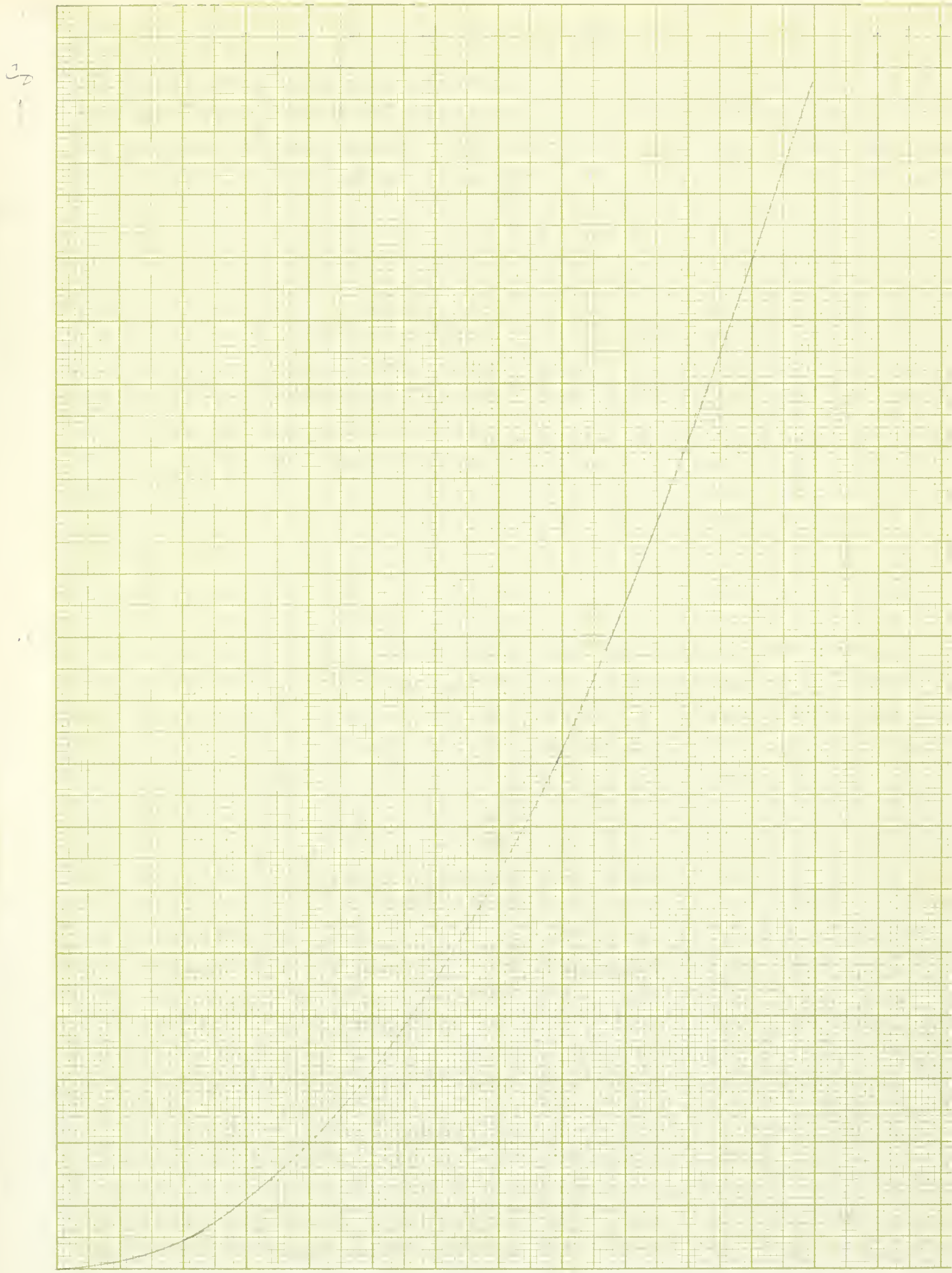
With best regards, I am,

Sincerely yours,

KOF/hf
Encl.

K. O. Friedrichs


α_1
α_2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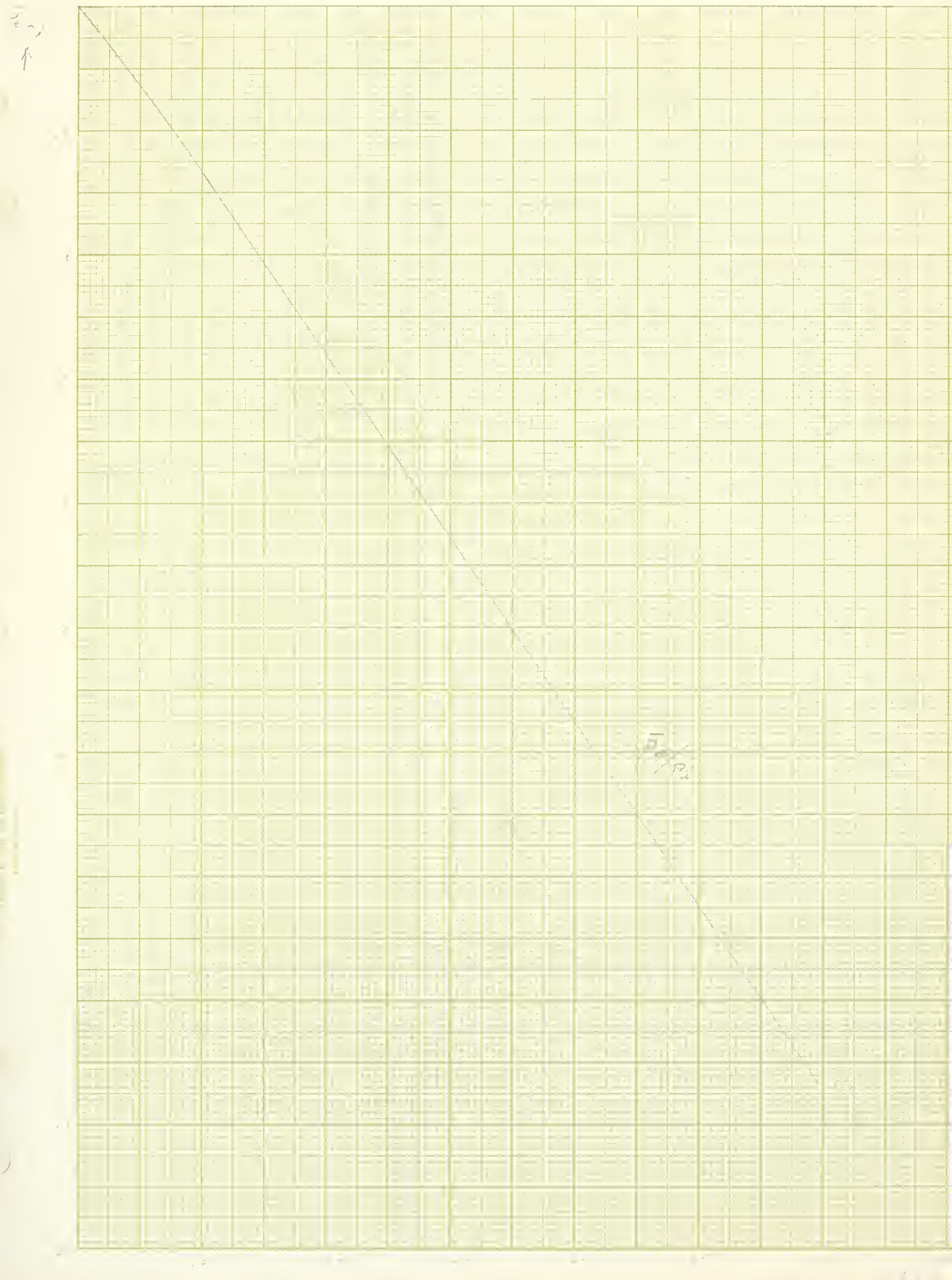
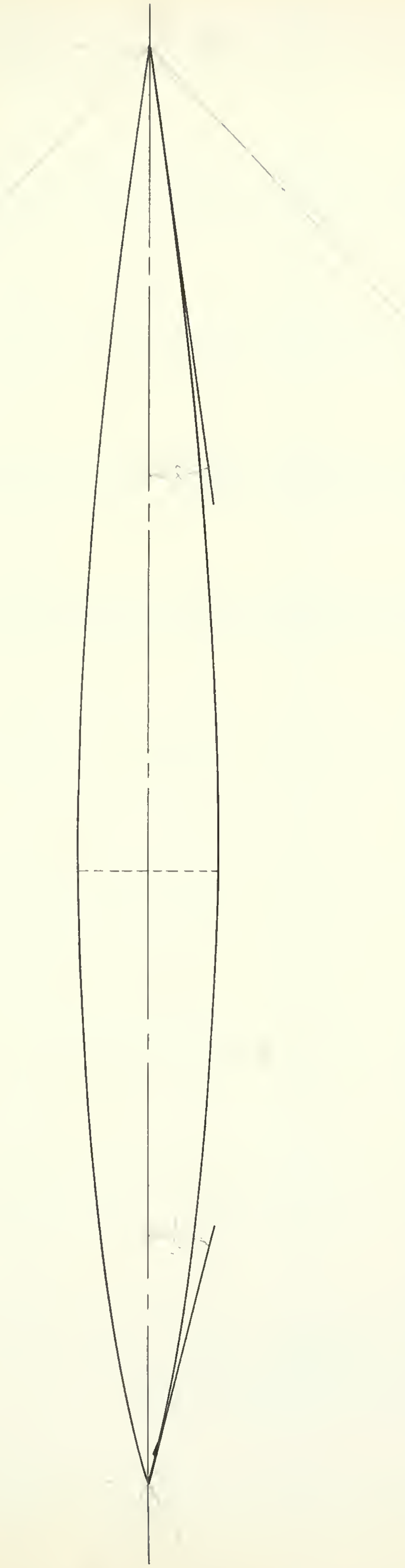

$\bar{P}_e$
P_i



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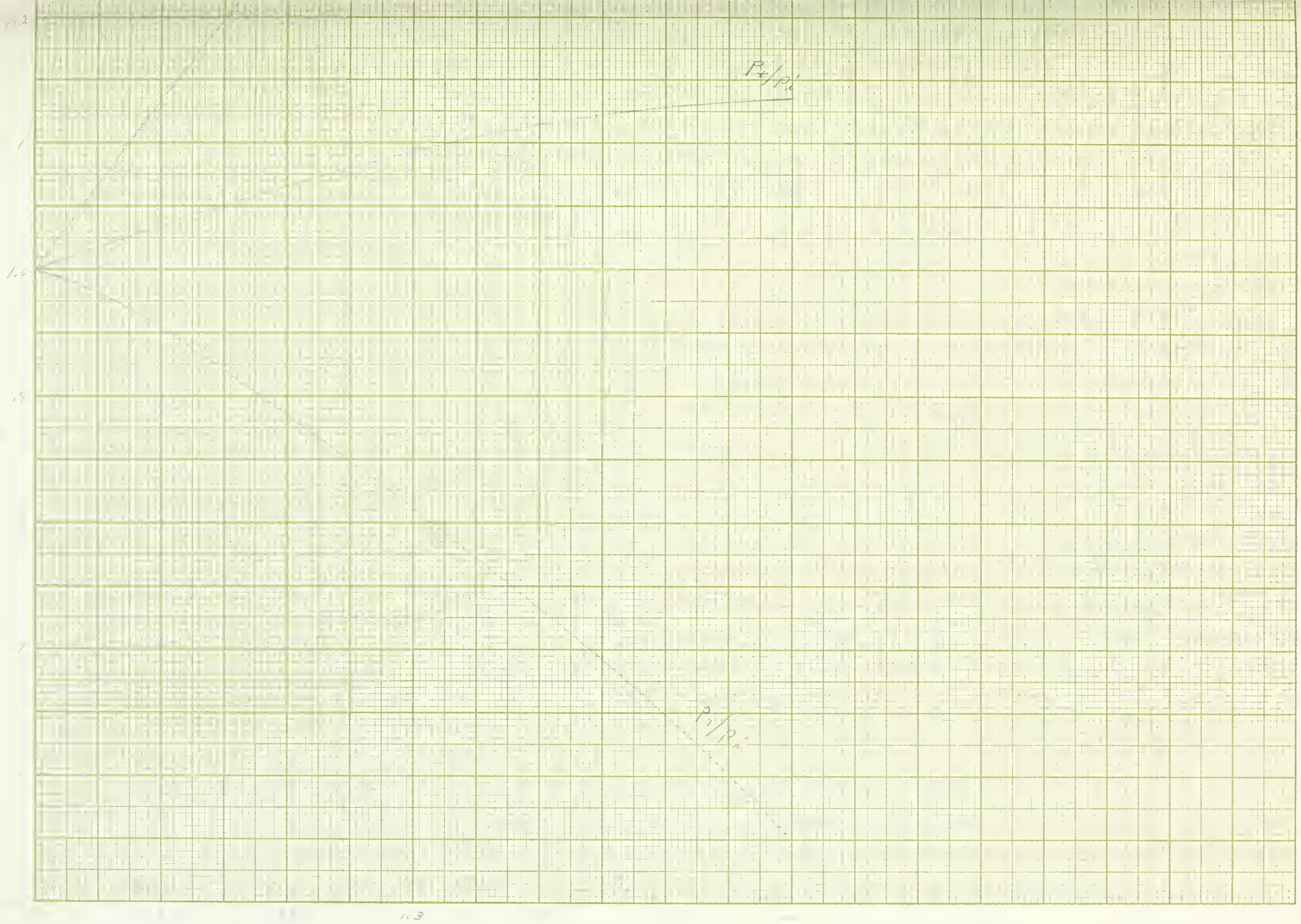
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